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Coincidence Theorems in Topological Spaces and Their Applications

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Abstract—Some new coincidence theorems involving new classes of set-valued mappings containing composites of acyclic mappings and the mappings having compactly local intersection property is proved in noncompact topological spaces. As applications, a new minimax inequality and a new section theorem are given in general topological spaces. © 1999 Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION AND PRELIMINARIES

Let X and Y be two nonempty sets. We denote by 2^Y and $\mathcal{F}(X)$ the family of all subsets of Y and the family of all nonempty finite subsets of X, respectively. Let Δ_n be the standard n-dimensional simplex with vertices $e_0, e_1, \dots, e_n$. If J is a nonempty subset of $\{0, 1, \dots, n\}$, we denote by Δ_J the convex hull of the vertices $\{e_j : j \in J\}$. A topological space X is said to be contractible if the identity mapping I_X of X is homotopic to a constant function. A topological space X is said to be an acyclic space if all of its reduced Cech homology groups over the rationals vanish. In particular, any contractible space is acyclic, and hence, any convex or star-shaped set in a topological vector space is acyclic. For a topological space X, we shall denote by $\mathrm{ka}(X)$ the family of all compact acyclic subsets of X. The following notions were introduced by Ding [1]. Let X be a topological space. A subset A of X is said to be compactly open (respectively, compactly closed) in X if for any nonempty compact subset K of X, $A \cap K$ is open (respectively, closed) in K. For any given subset A of X, we define the compact closure and the compact interior of A, denoted by $\mathrm{ccl}(A)$ and $\mathrm{cint}(A)$ as

$$\mathrm{ccl}(A) = \bigcap \{B \subset X : A \subset B \text{ and } B \text{ is compactly closed in } X\} \text{ and}$$
$$\mathrm{cint}(A) = \bigcup \{B \subset X : B \subset A \text{ and } B \text{ is compactly open in } X\},$$

respectively. It is easy to see that for each nonempty compact subset K of X, we have $\mathrm{ccl}(A) \cap K = \mathrm{cl}_K(A \cap K)$ and $\mathrm{cint}(A) \cap K = \int_K(A \cap K)$ where $\mathrm{cl}_K(A \cap K)$ and $\int_K(A \cap K)$ denote the closure

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and the interior of $A \cap K$ in K, respectively. It is clear that a subset A of X is compactly open (respectively, compactly closed) in X if and only if $\operatorname{cint}(A) = A$ (respectively, $\operatorname{ccl}(A) = A$). Let X and Y be two topological spaces. A mapping $G : X \to 2^Y$ is said to be transfer compactly open-valued (respectively, transfer compactly closed-valued) on X if for $x \in X$ and for each nonempty compact subset K of X, $y \in G(x) \cap K$ (respectively, $y \notin G(x) \cap K$) implies that there exists a point $x' \in X$ such that $y \in \int_K (G(x') \cap K)$ (respectively, $y \notin \operatorname{cl}_K(G(x') \cap K)$). Clearly, each open-valued (respectively, closed-valued) mapping G is transfer open-valued (respectively, transfer closed-valued) (see the definitions 6 and 7 in [2]) and is also compactly open-valued (respectively, compactly closed-valued). Each transfer open-valued (respectively, transfer closed-valued) mapping G is transfer compactly open-valued (respectively, transfer compactly closed-valued) and the inverse is not true in general.

Let X and Y be two topological spaces. A mapping $G : X \to 2^Y$ is said to have the local intersection property on X if for each $x \in X$ with $G(x) \neq \emptyset$, there exists an open neighborhood $N(x)$ of x in X such that $\bigcap_{z \in N(x)} G(z) \neq \emptyset$ (see [3]). The example in [3, p. 63] shows that a set-valued with the local intersection property may not have the property of open inverse values. Now, we introduce the following new notion for set-valued mappings. $G : X \to 2^Y$ is said to have the compactly local intersection property on X if for each nonempty compact subset K of X and for each $x \in K$ with $G(x) \neq \emptyset$, there exist a open neighborhood $N(x)$ of x in X such that $\bigcap_{z \in N(x) \cap K} G(z) \neq \emptyset$. Clearly, each set-valued mapping with the local intersection property have the compactly local intersection property and the inverse is not true in general. A mapping $G : X \to 2^Y$ is compact provided $G(X)$ is contained in a compact subset of Y.

Let X and Y be two topological spaces. For a given class $\mathbf{L}$ of set-valued mappings, define

$$\mathbf{L}(X, Y) = \{T : X \to 2^Y \mid T \in \mathbf{L}\}, \qquad \mathbf{L}_c = \{T = T_m T_{m-1} \dots T_1 \mid T_i \in \mathbf{L}\}.$$

Using the above notation we have the following definitions.

(1) T is an acyclic mapping, written $F \in \mathbf{V}(X, Y)$, if $F : X \to \operatorname{ka}(Y)$ is upper semicontinuous.
(2) $T \in \mathbf{V}^+(X, Y)$ if for any σ-compact subset K of X there exists a $T^* \in \mathbf{V}(K, Y)$ such that $T^*(x) \subset T(x)$ for all $x \in K$.
(3) $T \in \mathbf{V}_c^+(X, Y)$ if for any σ-compact subset K of X there exists a $T^* \in \mathbf{V}_c(K, Y)$ such that $T^*(x) \subset T(x)$ for all $x \in K$.

In 1998, Ding [4] established the following coincidence theorem which generalizes the corresponding results in [5–9].

THEOREM A. *Let X be a Hausdorff topological space and D a contractible subset of a topological space Y. Let $S : D \to 2^X$ and $T : X \to 2^D$ be such that*

(i) *$S \in \mathbf{V}_c^+(D, X)$ is a compact mapping,*
(ii) *for each $x \in X$, $T(x) \neq \emptyset$ and T has the local intersection property,*
(iii) *for each open set $U \subset X$, the set $\bigcap_{x \in U} T(x)$ is empty or contractible.*

Then there exists $(x_0, y_0) \in X \times D$ such that $x_0 \in S(y_0)$ and $y_0 \in T(x_0)$.

In this paper, we shall first establish the relationship between the compactly local intersection property and the transfer compactly open-valued property for a set-valued mapping. Next, some new coincidence theorems involving a new class of mappings containing composites of acyclic mappings defined in general topological spaces are proved where these mappings may not have convex values and open inverse values, and their domains may not be compact. The local intersection property of mappings is further relaxed. These theorems further generalize the corresponding results in [4–9]. As applications, a new minimax inequality and a new section theorem are obtained in topological spaces. These theorems improve and generalize the corresponding results in [10–13].

LEMMA 1.1. *Let X and Y be topological spaces and $G : X \to 2^Y$ a set-valued mapping with nonempty values. Then the following conditions are equivalent:*

(I) *T has the compactly local intersection property,*

(II) *for each compact subset K of X and for each $y \in Y$, there exists a open subset O_y of X (which may be empty) such that $O_y \cap K \subset G^{-1}(y)$ and $K = \bigcup_{y \in Y}(O_y \cap K)$,*

(III) *for any compact subset K of X, there exists a set-valued mapping $F : X \to 2^Y$ such that $F(x) \subset G(x)$ for each $x \in X$, $F^{-1}(y)$ is open in X and $F^{-1}(y) \cap K \subset G^{-1}(y)$ for each $y \in Y$, and $K = \bigcup_{y \in Y}(F^{-1}(y) \cap K)$,*

(IV) *for each compact subset K of X and for each $x \in K$, there exists $y \in Y$ such that $x \in \int(G^{-1}(y)) \cap K$ and*

$$K = \bigcup_{y \in Y} \int_K (G^{-1}(y) \cap K),$$

(V) *$G^{-1} : Y \to 2^X$ is transfer compactly open-valued on X.*

PROOF.

(I) $\Rightarrow$ (II). By (i), for each compact subset K of X and for each $x \in K$, with $G(x) \neq \emptyset$, and hence, there exists an open neighborhood $N(x)$ of x in X such that

$$M(x) = \bigcap_{z \in N(x) \cap K} G(z) \neq \emptyset.$$

It follows that there exists $y \in M(x) \subset Y$ such that $N(x) \cap K \subset G^{-1}(y)$, and hence, we have

$$K = \bigcup_{x \in K} (N(x) \cap K) \subset \bigcup_{y \in Y} (G^{-1}(y) \cap K) = K.$$

For each $y \in Y$, if $y \in M(x)$ for some $x \in K$, let $O_y = N(x)$ and if $y \notin M(x)$ for all $x \in K$, let $O_y = \emptyset$. Then the family $\{O_y\}_{y \in Y}$ of open sets satisfies Condition (II).

(II) $\Rightarrow$ (III). Suppose Condition (II) holds. Define a mapping $F : X \to 2^Y$ by

$$F(x) = \{y \in Y : x \in O_y\}, \qquad \forall x \in X.$$

Then for each $y \in Y$, we have

$$F^{-1}(y) = \{x \in X : y \in F(x)\} = \{x \in X : x \in O_y\} = O_y \subset G^{-1}(y)$$

and

$$K = \bigcup_{y \in Y} (O_y \cap K) = \bigcup_{y \in Y} (F^{-1}(y) \cap K).$$

(III) $\Rightarrow$ (IV). Suppose Condition (III) holds. Then for each $y \in Y$, $F^{-1}(y) \subset \int(G^{-1}(y)) \subset G^{-1}(y)$. Therefore, for each compact subset K of X, we have

$$K = \bigcup_{y \in Y} (F^{-1}(y) \cap K) \subset \bigcup_{y \in Y} \left(\int (G^{-1}(y)) \cap K\right) \subset \bigcup_{y \in Y} \int_K (G^{-1}(y) \cap K) \subset K,$$

and for each $x \in K$, there exists $y \in Y$ such that $x \in \int(G^{-1}(y)) \cap K$.

(IV) $\Rightarrow$ (V). Suppose Condition (IV) holds. Then for each compact subset K of X,

$$K = \bigcup_{y \in Y} \int_K (G^{-1}(y) \cap K) = \bigcup_{y \in Y} (G^{-1}(y) \cap K).$$

It follows that for each $y \in Y$, $x \in G^{-1}(y) \cap K$ implies that there exists a point $y' \in Y$ such that $x \in \int_K (G^{-1}(y') \cap K)$. This shows that G^{-1} is transfer compactly open-valued on Y.

(V) $\Rightarrow$ (I). Suppose Condition (V) holds. We first show that for each compact subset K of X,

$$\bigcup_{y\in Y} \int_K (G^{-1}(y) \cap K) = \bigcup_{y \in Y} (G^{-1}(y) \cap K).$$

It suffices to show $\bigcup_{y\in Y}(G^{-1}(y) \cap K) \subset \bigcap_{y\in Y} \int_K (G^{-1}(y) \cap K)$. If it is false, then there exists $x \in \bigcup_{y\in Y}(G^{-1}(y) \cap K)$ such that $x \notin \bigcup_{y \in Y} \int_K (G^{-1}(y) \cap K)$. Hence, there exists $y \in Y$ such that $x \in \int_K (G^{-1} \cap K)$ such that $y \notin \int_K (G^{-1}(z) \cap K)$ for all $z \in Y$. Since G^{-1} is transfer compactly open-valued on Y, there must be a $y' \in Y$ such that $x \in \int_K (G^{-1}(y') \cap K)$ which is a contradiction. Hence, noting $G(x) \neq \emptyset$ for each $x \in X$, we have

$$K = \bigcap_{y \in Y} (G^{-1}(y) \cap K) = \bigcup_{y \in Y} \int_K (G^{-1}(y) \cap K).$$

It follows that for each $x \in X$, there exists $y \in Y$ such that $x \in \int_K (G^{-1}(y) \cap K)$ and so there is a relatively open neighborhood $N_1(x)$ of x in K such that $N_1(x) \subset \int_K (G^{-1}(y) \cap K)$. Therefore, there exists an open neighborhood $N(x)$ of x in X such that

$$N(x) \cap K = N_1(x) \subset \int_K (G^{-1}(y) \cap K) \subset G^{-1}(y).$$

It follows that $y \in \bigcap_{z \in N(x) \cap K} G(x)$ and $\bigcap_{z \in N(x)\cap K} G(z) \neq \emptyset$. This shows that G has the compactly local intersection property on X.

The following result is contained in the proof of Theorem 1 in [14] (see also, [15]).

LEMMA 1.2. *Let Y be a topological space. For any nonempty subset J of $\{0, 1, \dots, n\}$, let Γ_J be a nonempty contractible subset of Y. If $\emptyset \neq J \subset J' \subset \{0, 1, \dots, n\}$ implies $\Gamma_J \subset \Gamma_{J'}$, then there exists a continuous mapping $f : \Delta_n \to Y$ such that $f(\Delta_J) \subset \Gamma_J$ for each nonempty subset J of $\{0, 1, \dots, n\}$.*

The following result is quite well known in the Lefschetz fixed point theory. For details, we refer the reader to [16,17].

LEMMA 1.3. *Let Δ_n be an n-dimensional simplex with the Euclidean topology. If $F \in \mathbf{V}_c(\Delta_n, \Delta_n)$ then F has a fixed point.*

2. COINCIDENCE THEOREMS

THEOREM 2.1. *Let X be a Hausdorff topological space and Y be a contractible space. Suppose the following conditions hold:*

(i) *$F \in \mathbf{V}_c^+(Y, X)$ is a compact mapping,*
(ii) *$G : X \to 2^Y$ is such that $G(x) \neq \emptyset$ for each $x \in X$ and one of Conditions (I)–(V) in Lemma 1.1 is satisfied,*
(iii) *for each compactly open subset U of X, $\bigcap_{x \in U} G(x)$ is empty or contractible.*

Then there exist $(x_0, y_0) \in X \times Y$ such that $x_0 \in F(y_0)$ and $y_0 \in G(x_0)$.

PROOF. By (i), $\overline{F(Y)}$ is compact in X. Noting that $G(x) \neq \emptyset$ for each $x \in X$ and Conditions (I)–(V) in Lemma 1.1 are equivalent, by (ii) and Lemma 1.1, for each $y \in Y$ there exists an open subset O_y of X such that $O_y \cap \overline{F(Y)} \subset G^{-1}(y)$ and

$$\overline{F(Y)} = \bigcup_{y \in Y} \left(O_y \cap \overline{F(Y)} \right).$$

Hence, there exists a finite set $\{y_0, y_1, \ldots, y_n\} \subset Y$ such that

$$\overline{F(Y)} = \bigcup_{i=0}^{n} \left(O_{y_i} \cap \overline{F(Y)}\right).$$

Now for each nonempty subset J of $N = \{0, 1, \ldots, n\}$, define

$$\Gamma_J = \begin{cases} \bigcap\{G(x) : x \in \bigcap_{j\in J} \left(O_{y_j} \cap \overline{F(Y)}\right)\}, & \text{if } \bigcap_{j\in J} \left(O_{y_j} \cap \overline{F(Y)}\right) \neq \emptyset, \\ Y, & \text{otherwise.} \end{cases}$$

Note that $O_y \cap \overline{F(Y)} \subset G^{-1}(y)$ for each $y \in Y$, if $x \in \bigcap_{j\in J}(O_{y_j} \cap \overline{F(Y)})$, then $\{y_j : j \in J\} \subset G(x)$. Obviously, each $\bigcap_{j\in j}(O_{y_j} \cap \overline{F(Y)})$ is a compactly open subset of X, by (iii), each Γ_J is nonempty contractible and it is clear that $\Gamma_J \subset \Gamma_{J'}$, whenever $\emptyset \neq J \subset J' \subset N$. By Lemma 1.2, there exists a continuous mapping $f : \Delta_n \to Y$ such that

$$f(\Delta_J) \subset \Gamma_J, \qquad \text{for all } J \subset N, \quad J \neq \emptyset.$$

Since $f(\Delta_n)$ is compact in Y and $F \in \mathbf{V}_c^+(Y, X)$, there exists a $\widetilde{F} \in \mathbf{V}_c(f(\Delta_n), X)$ such that $\widetilde{F}(y) \subset F(y)$ for each $y \in f(\Delta_n)$. By (2.1), we have

$$\widetilde{F}(f(\Delta_n)) = \bigcup_{i=0}^{n} \left[\left(O_{y_i} \cap \widetilde{F}(f(\Delta_n))\right]\right..$$

Let $\{\psi_i\}_{i\in N}$ be a continuous partition of unity subordinated to the open covering $\{O_{y_i} \cap \widetilde{F}(f(\Delta_n))\}_{i\in N}$, i.e., for each $i \in N$, $\psi_i : \widetilde{F}(f(\Delta_n)) \to [0,1]$ is continuous,

$$\left\{x \in \widetilde{F}(f(\Delta_n)) : \psi_i(x) \neq 0\right\} \subset O_{y_i} \cap \widetilde{F}(f(\Delta_n)) \subset O_{y_i} \subset G^{-1}(y_i),$$

such that $\sum_{i=0}^{n} \psi_i(x) = 1$ for all $x \in \widetilde{F}(f(\Delta_n))$. Define $\psi : \widetilde{F}(f(\Delta_n)) \to \Delta_n$ by

$$\psi(x) = (\psi_0(x), \psi_1(x), \ldots, \psi_n(x)), \qquad \text{for all } x \in \widetilde{F}(f(\Delta_n)).$$

Then $\psi(x) \in \Delta_{J(x)}$ for all $x \in \widetilde{F}(f(\Delta_n))$, where $J(x) = \{j \in N : \psi_j(x) \neq 0\}$. Therefore, we have

$$f(\psi(x)) \in f\left(\Delta_{J(x)}\right) \subset \Gamma_{J(x)} \subset G(x), \qquad \text{for all } x \in \widetilde{F}(f(\Delta_n)).$$

It is easy to see $\psi \circ \widetilde{F} \circ f \in \mathbf{V}_c(\Delta_n, \Delta_n)$, by Lemma 1.3, there exists $z \in \Delta_n$ such that $z \in \psi \circ \widetilde{F} \circ f(z)$. Let $y_0 = f(z)$, then $y_0 \in f(\Delta_n) \subset Y$ and $\psi^{-1}(z) \cap \widetilde{F}(y_0) \neq \emptyset$. Take $x_0 \in \psi^{-1}(z) \cap \widetilde{F}(y_0)$, then we have $z = \psi(x_0)$ and $x_0 \in \widetilde{F}(y_0) \subset F(y_0)$. It follows from (2.2) that

$$y_0 = f(z) = f(\psi(x_0)) \subset G(x_0).$$

COROLLARY 2.1. *Let X be a compact Hausdorff topological space and Y be a contractible space. Let $F \in \mathbf{V}_c^+(Y, X)$ and $G : X \to 2^Y$ with nonempty values such that,*

(i) *G satisfies one of the Conditions (I)–(V) in Lemma 1.1,*
(ii) *for each open set $U \subset X$, the set $\bigcap_{x\in U} G(x)$ is empty or contractible.*

Then there exists $(x_0, y_0) \in X \times Y$ such that $x_0 \in F(y_0)$ and $y_0 \in G(x_0)$.

PROOF. Since X is compact, It is easy to see that if $F(Y)$ is replaced by X in the proof of Theorem 2.1, the proof is still valid, and hence, the conclusion of Corollary 2.1 follows.

REMARK 2.1. Theorem 2.1 improves Theorem 1 in [4] by relaxing the local intersection property of G. Corollary 2.1 improves Theorem 2 in [4], Theorem 1 in [9] and Theorem 1 in [6] in the following aspects:

(1) X may not be compact space,
(2) F is a mapping in the new class of mappings containing the composites of acyclic mappings,
(3) G may not have the local intersection property and the property of open inverse values.

Theorem 2.1, in turn, generalizes Theorem 1 in [5], Theorem 4.5 in [7], and Theorem 1 in [18].

3. SOME APPLICATIONS

THEOREM 3.1. *Let X be a Hausdorff topolocial space and Y a contractible. Suppose the functions $f, g : X \times Y \to \mathbf{R}$ such that*

(i) *$f(x,y) \leq g(x,y)$ for all $(x,y) \in X \times Y$,*
(ii) *for any $\alpha \in \mathbf{R}$, the mapping $G : X \to 2^Y$ defined by $G(x) = \{y \in Y : f(x,y) > \alpha\}$ satisfies one of the Conditions (I)–(V) in Lemma 1.1,*
(iii) *for each compact open subset U of X, the set $\bigcap_{x \in U} G(x)$ is empty or contractible,*
(iv) *for each $K \in C(X)$, $y \in Y$ and $r \in \mathbf{R}$, the set $\{x \in K : g(x,y) \leq r\} \in \mathrm{ka}(X)$ where $C(X)$ is the family of all nonempty compact subsets of X.*

Then

$$\inf_{x \in X} \sup_{y \in Y} f(x,y) \leq \inf_{K \in C(X)} \sup_{y \in Y} \inf_{x \in K} g(x,y).$$

PROOF. If the conclusion is not true, there exist $\alpha, \beta \in \mathbf{R}$ such that

$$\inf_{x \in X} \sup_{y \in Y} f(x,y) > \alpha > \beta > \inf_{K \in C(X)} \sup_{y \in Y} \inf_{x \in K} g(x,y).$$

Hence, there exist a $K \in C(X)$ such that $\beta > \sup_{y \in Y} \inf_{x \in K} g(x,y)$. Define mappings $F : Y \to 2^X$ and $G : X \to 2^Y$ by

$$F(y) = \{x \in K : g(x,y) \leq \beta\} \quad \text{and} \quad G(x) = \{y \in Y : f(x,y) > \alpha\}.$$

Then $F(y) \neq \emptyset$ for each $y \in Y$ and by (iv), $F \in \mathbf{V}(Y,K) \subset \mathbf{V}_c^+(Y,K)$ is a compact mapping. Clearly, $G(x) \neq \emptyset$ for each $x \in X$ and by (ii) and (iii), all conditions of Theorem 2.1 are satisfied. By Theorem 2.1, there exist $(x_0, y_0) \in K \times Y$ such that $x_0 \in F(y_0)$ and $y_0 \in G(x_0)$. It implies

$$g(x_0, y_0) \leq \beta < \alpha < f(x_0, y_0).$$

This contradicts Assumption (i). This completes the proof.

REMARK 3.1. Theorem 3.1 improves and develops Theorem 5 in [10] and Theorem 4 in [11] in several aspects and the method of proof differs from that in [10,11].

THEOREM 3.2. *Let X be a Hausdorff topological space and Y a contractible space. Let M, N be two subsets of $X \times Y$ such that*

(i) *the mapping $G, H : X \to 2^Y$ defined by*

$$G(x) = \{y \in Y : (x,y) \notin M\} \quad \text{and} \quad H(x) = \{y \in Y : (x,y) \notin N\}$$

is such that $G(x) \subset H(x)$ for each $x \in X$ and G satisfies one of the Conditions (I)–(V) in Lemma 1.1,
(ii) *for each compactly open set $U \subset X$, the set $\bigcap_{x \in U} G(x)$ is empty or contractible,*
(iii) *there exist a closed subset Q of $X \times Y$ with $Q \subset N$ and a compact subset K of X such that for each $y \in Y$, the set $\{x \in K : (x,y) \in Q\}$ is acyclic.*

Then there exists a point $x_0 \in K$ such that $\{x_0\} \times Y \subset M$.

PROOF. Define a mapping $F : Y \to 2^K$ by $F(y) = \{x \in K : (x,y) \in Q\}$. Since Q is closed in $X \times Y$ and K is compact, F has a closed graph, and hence, F is upper semicontinuous and $F \in \mathbf{V}(Y,K) \subset \mathbf{V}_c^+(Y,K)$. Suppose that $G(x) \neq \emptyset$ for each $x \in K$. Then the restriction $G|_K$ of G on K and F satisfy all conditions of Corollary 2.1. By Corollary 2.1, there exists $(x_0, y_0) \in K \times Y$ such that $x_0 \in F(y_0)$ and $y_0 \in G(x_0) \subset H(x_0)$, i.e., $(x_0, y_0) \in Q \subset N$ and $(x_0, y_0) \notin N$. This is a contradiction. Therefore, there exists a point $x_0 \in K$ such that $G(x_0) = \emptyset$ and hence $\{x_0\} \times Y \subset M$. This completes the proof.

REMARK 3.2. Theorem 3.2 improves and generalizes Theorem 2 in [10] and Theorem 3 in [11] in several aspects, and in turn generalizes Fan's section theorems in [12,13].

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